

# CONICALLY CAMBERED DELTA WINGS IN SUPERSONIC FLOW PART I - BASIC SOLUTIONS

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## ABSTRACT

The analysis of a conically cambered delta wing with subsonic leading edges in a supersonic stream is presented. The twist distribution chosen is a polynomial of even ordered terms in the conical coordinate for which a closed form series solution is obtained. It is found that the mean camber shapes are geometrically simple and even in their unoptimized form appear to be quite promising in terms of low drag. To avoid flow separation at the leading edges at the design condition, the attachment line is prescribed there. This results in a leading edge droop for all the basic shapes.

### Nomenclature

$a$	$= (M^2 - 1)^{1/2} \tan \delta$ , slenderness parameter
AR	aspect ratio
$B(\nu, \mu)$	beta function with arguments $\nu$ and $\mu$
$C_D$	drag coefficient
$C_L$	lift coefficient
$C_M$	pitching moment coefficient
$C_o$	a constant proportional to the leading edge singularity
$\Delta C_p$	net pressure coefficient
$C_r$	root chord
$e$	$= \pi AR C_D / C_L^2$ , lift dependent drag factor
E	complete elliptic integral of second kind
$F(a, \beta, \gamma, \delta)$	hypergeometric function with arguments $a, \beta, \gamma$ and $\delta$
$k$	$= (1 - a^2)^{1/2}$
K	complete elliptic integral of first kind

L	lift
$L(\eta)$	spanwise distribution of lift
M	free stream Mach number
s	local semispan
u, w	x- and z-components of perturbation velocity
U	free stream velocity
x, y, z	Cartesian coordinates, Fig. 1.
Z, $\Delta Z$	wing ordinates

### Greek Symbols

$\alpha, \alpha_0$	angle of attack distribution, root incidence
$\delta$	wing semi-apex angle
$\epsilon_{0k}, \epsilon_{2k}$	constant associated with the twist distribution $\eta^{2k}/2k$
$\eta$	conical coordinate, $y/s(x)$

### Introduction

The problem of wings of arbitrary planform with completely subsonic leading edges in a supersonic flow has not been solved analytically. The plane delta wing was solved by Etkin and Woodward<sup>1</sup> using an approximate method based on the ideas of Evvard<sup>2</sup> and Krasilchikova<sup>3</sup>. However, elegant and closed form solutions for the delta wing may be obtained using conical flow theory, originally due to Busemann<sup>4</sup>.

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Early investigators using conical flow assumptions also made the low aspect ratio assumption, thereby rendering their solutions independent of the wave drag. Both the direct and the indirect lifting problems were studied. An early attempt of Jones<sup>5</sup> led to his famous low aspect ratio theory which gave a simple analytical solution. Without the low aspect ratio assumption Shaw<sup>6</sup>, in his studies of leading edge controls on a delta wing, gave the solution for the case when the central portion of the wing is straight and the outboard portion drooped. The twist distribution he used was of the type

$$\begin{aligned} w/v &= c & 0 \leq |\eta| \leq \eta_0 \\ &= c + d(\eta - \eta_0)^m, & \eta_0 < |\eta| \leq 1 \end{aligned} \quad (1)$$

with  $m = 0$ . Here  $w$  is the  $z$ -perturbation velocity,  $U$  the free-stream velocity,  $\eta$  the conical coordinate,  $c$ ,  $d$ ,  $m$  and  $\eta_0$  are constants. For slender wings this was extended by Brebner<sup>7</sup> to cases when  $m = 1$  and  $2$ , and by Weber<sup>8</sup>, who in addition, considered the constants  $C$ ,  $d$  and  $\eta_0$  as functions of the streamwise coordinate  $x$ . Weber was therefore able to extend the method to cases of arbitrary planform and downwash distribution for very slender wings. More recently, the first author has given simple closed form solutions for a slender delta wing with a polynomial twist distribution used in this paper<sup>9</sup>.

In the above theories<sup>5-9</sup> the surface boundary condition was satisfied on the  $z = 0$  plane. Using a slightly refined procedure, Smith<sup>10</sup> solved for the case when the spanwise camber line is a circular arc, by satisfying the boundary condition on the wing camber surface. Cooke<sup>11</sup> in the same spirit extended it to a camber line with a straight central portion and drooped edges. Smith's results come out as a special case.

The direct problem of specifying the load distribution and determining the spanwise camber was solved by Smith and Mangler<sup>12</sup>, again using the low aspect ratio assumption and satisfying the surface boundary condition on the plane  $z = 0$ . They could relate the twist distributions for different values of the slenderness parameter, but carrying the same spanwise distribution of lift, hence considerable attention was paid to the slender wing solution for which closed form solutions could be obtained.

However, the drawbacks of any low aspect ratio theory are well known. Therefore, the delta wing problem without this assumption has been attempted, among others by Baldwin<sup>13</sup>, and Tsien<sup>14</sup> for the direct problem and by Holla<sup>15</sup>, Holla, et.al.<sup>16</sup>, and the present paper for the in-

direct lifting problem. Holla, on the recommendation of Carafoli<sup>18</sup>, has used a twist distribution of the type  $(1 - a^2 \eta^2)^{3/2} \eta^{2m}$  with positive integral values of  $m$ . This distribution is dependent on the free-stream Mach number and the wing geometry through the slenderness parameter  $a$ . This is an unnecessary restriction, since for a given wing one would like to express its geometry in terms of basic twist distributions which are independent of the Mach number. He, further, could not obtain a general solution for arbitrary values of  $m$  and for practical reasons was restricted to  $m = 1$  to  $5$ . This was later extended upto  $m = 8$  in Ref. 17. However, the final results, and in particular the expressions for drag, are lengthy and cumbersome.

In this paper twist distributions for the type  $\eta^{2m}$  for positive integral values of  $m$  are considered. The pressure distributions are obtained in a series solution in increasing powers of the slenderness parameter,  $a$ . The general simplicity of the results are appealing. The case when the twist distribution is proportional to  $|\eta|$  was given by Carafoli<sup>18</sup>.

In order to avoid flow separation at the leading edges, most authors<sup>7-18</sup> have prescribed an attachment line at the leading edge. Since this is a useful practical design consideration at cruise, we have also included it in our study. Some experiments on conical wings are reported in Refs. 19-21 which generally show that it is possible to obtain zero load condition at the leading edge and that the conical pressure distribution in such cases helps the flow to stay attached at the leading edge at higher incidences. This along with the fact, that delta wings are generally easier to manufacture, have high wing thickness near the root chord even for small thickness to chord ratios, and greater structural rigidity, makes the study of conically cambered delta wings useful.

### Analysis

The wing-Mach cone system is shown in Fig. 1. Throughout our analysis, the leading edges are subsonic and the wing is at zero yaw. For this case Carafoli<sup>18</sup> has derived a relationship between the  $u$  and  $w$  components of velocity on the wing surface which reads

$$\begin{aligned} u(\eta) &= \frac{UC_0}{\tan \delta \sqrt{1 - \eta^2}} + \frac{U \tan \delta}{\pi} \int \frac{dw(\eta_i)}{d\eta_i} \frac{\eta_i}{\sqrt{1 - a^2 \eta_i^2}} \\ &\quad \times \ln \left( \frac{\sqrt{1 - \eta^2} + \sqrt{1 - \eta_i^2}}{\sqrt{1 - \eta^2} - \sqrt{1 - \eta_i^2}} \right) d\eta_i \end{aligned} \quad (2)$$



and

$$\begin{aligned}
 -w(o) = & \frac{C_o E(k)}{\tan^2 \delta} + \frac{2}{\pi} a^2 K(k) \int_0^1 \frac{dw}{d\eta} \frac{\eta \sqrt{1-\eta^2}}{\sqrt{1-a^2 \eta^2}} d\eta \\
 & + \int_0^1 \frac{dw}{d\eta} d\eta - \frac{2}{\pi} K(k) \int_0^1 \frac{dw}{d\eta} \int_0^\phi (1-a^2 \sin^2 \phi)^{1/2} \\
 & d\phi d\eta + \frac{2}{\pi} [K(k) - E(k)] \int_0^1 \frac{dw}{d\eta} \\
 & \int_0^\phi (1-a^2 \sin^2 \phi)^{-1/2} d\phi d\eta \quad (3)
 \end{aligned}$$

where the new symbols appearing are  $\delta$  the wing semi-apex angle,  $a$  the slenderness parameter ( $= \sqrt{\mu^2 - 1} \tan \delta$ ),  $M$  the Mach number,  $u$  the x-perturbation velocity,  $k = \sqrt{1-a^2}$ ,  $K$  and  $E$  are complete elliptic integrals of the first and second kind, respectively.

The net pressure coefficient  $\Delta C_p$  at any point is

$$\Delta C_p = 4\mu/U \quad (4)$$

In Eq. (1) the first term on the right is singular and corresponds to a plane delta wing. The second term is the  $\Delta C_p$  distribution that vanishes at the leading edges. For a given wing and Mach number, zero load at the leading edge is realised (attachment line at leading edge) when  $C_o = 0$ . This will be so only at one incidence for a given Mach number.

For arbitrary  $w(\eta)$ , difficulties arise because of the logarithmic singularity and the term  $(1-a^2 \eta^2)^{-1/2}$  when  $a = 1$  (sonic leading edges). In further analyses we restrict ourselves to cases where  $a < 1$  and use a twist distribution of the type

$$w(\eta) = \epsilon_o + \sum_{m=1}^{\infty} \epsilon_{2m} \eta^{2m/2m} \quad (5)$$

where  $\epsilon_o$  and  $\epsilon_{2m}$  are constants.

On substituting for  $w(\eta)$  in Eq. (1) and expanding the term  $(1-a^2 \eta^2)^{-1/2}$  in a binomial series

$$(1-a^2 \eta^2)^{-1/2} = \sum_{n=0}^{\infty} J_{2n} a^{2n} \eta^{2n}, \quad a < 1 \quad (6)$$

where  $J_0 = 1$

$$J_{2n} = \frac{1.3 \dots (2n-1)}{2.4 \dots (2n)} \quad (7)$$

one obtains after an integration by parts and the use of Glauert integrals

$$\begin{aligned}
 \Delta C_p(\eta) = & \frac{4C_o}{\tan \delta \sqrt{1-\eta^2}} \\
 & + \frac{4 \tan \delta}{\pi} \sum_{n=1}^{\infty} \epsilon_{2n} \sum_{m=0}^{\infty} J_{2m} a^{2m} I_{2(m+n)} \quad (8)
 \end{aligned}$$

where

$$I_{2k} = \frac{\pi}{2k+1} (1-\eta^2)^{1/2} \sum_{i=0}^k J_{2i} \eta^{2(k-i)} \quad (9)$$

The constant  $C_o$  in Eq. (2), which determines the strength of the leading edge singularity, for the twist distribution (5) is given by

$$\begin{aligned}
 C_o = & \frac{\tan^2 \delta}{E(k)} \left\{ -\epsilon_o + \left\langle \frac{2}{\pi} K(k) [E(a) - K(a)] \right. \right. \\
 & + \frac{2}{\pi} E(k) K(a) - 1 \left. \right\rangle \sum_{n=1}^{\infty} \frac{\epsilon_{2n}}{2n} \\
 & - \sum_{n=1}^{\infty} \frac{\epsilon_{2n}}{n\pi} [(2na^2 R_{2n} + S_{2n} - T_{2n}) K(k) + T_{2n} E(k)] \left. \right\} \quad (10)
 \end{aligned}$$

where

$$\begin{aligned}
 R_{2n} = & \int_0^1 \eta^{2n} \sqrt{\frac{1-\eta^2}{1-a^2 \eta^2}} d\eta \\
 = & \frac{1}{2} B\left(\frac{2n+1}{2}, \frac{3}{2}\right) F\left(\frac{2n+1}{2}, \frac{1}{2}; n+2; a^2\right) \quad (11)
 \end{aligned}$$

$$\begin{aligned}
 S_{2n} = & \int_0^1 \eta^{2n} \sqrt{\frac{1-a^2 \eta^2}{1-\eta^2}} d\eta \\
 = & \frac{1}{2} B\left(\frac{2n+1}{2}, \frac{1}{2}\right) F\left(\frac{2n+1}{2}, -\frac{1}{2}; n+1; a^2\right) \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 T_{2n} = & \int_0^1 \eta^{2n} \frac{d\eta}{\sqrt{1-a^2 \eta^2} \sqrt{1-\eta^2}} \\
 = & \frac{1}{2} B\left(\frac{2n+1}{2}, \frac{1}{2}\right) F\left(\frac{2n+1}{2}, \frac{1}{2}; n+1; a^2\right) \quad (13)
 \end{aligned}$$

and  $B$  and  $F$  are the complete Beta function and the hypergeometric function respectively.

The spanwise distribution of lift  $L(\eta)$  is given

by

$$\begin{aligned}
 L(\eta) = & \frac{1}{C_r C_L} \int_{x_1}^1 \Delta C_p(\eta_i) dx \\
 = & \frac{\eta}{C_r C_L} \int_{\eta}^1 \Delta C_p(\eta_i) \frac{d\eta_i}{\eta_i^2} \quad (14)
 \end{aligned}$$

On substituting for  $\Delta C_p$  one finally obtains

$$L(\eta) = \frac{1}{C_{rCL}} \left[ \frac{4 C_0 \sqrt{1-\eta^2}}{\tan \delta} + \frac{4 \tan \delta}{\pi} \sum_{n=1}^{\infty} \epsilon_{2n} \sum_{m=0}^{\infty} J_{2m} a^{2m} Q_{2(m+n)} \right] \quad (15)$$

where

$$Q_{2k} = \eta \int_{\eta}^1 \frac{l_{2k}}{\eta_i^2} d\eta_i = \frac{\pi(1-\eta^2)^{3/2}}{2k(2k+1)} \left[ \eta^{2(k-1)} + \sum_{m=1}^{k-1} (2m+1) J_{2m} \eta^{2(k-m-1)} \right] \quad (16)$$

For the mean camber shape at any chordwise station, we start from

$$\frac{\partial Z}{\partial x} = w(\eta) = Z - \eta \frac{\partial \bar{Z}}{\partial \eta} \quad (17)$$

where

$$Z = x \bar{Z}(\eta) \quad (18)$$

Integrating Eq.(17) we have

$$\bar{Z}(\eta) = H\eta - \eta \int_{\eta}^1 \frac{w(\eta)}{\eta^2} d\eta \quad (19)$$

where  $H$  is an arbitrary constant. This shows that the wing shapes are non-unique to the extent that a dihedral angle does not affect the aerodynamic characteristics in the linearized theory. Therefore, without any loss of generality, we put  $H = 0$ .

On substituting for  $w(\eta)$  in Eq. (19) we have

$$\bar{Z}(\eta) = - \sum_{k=1}^{\infty} \epsilon_{2k} \eta^{2k} / 2k(2k-1) - \epsilon_0 \quad (20)$$

If the ordinates are measured with respect to the wing median surface, we obtain using Eq.(18)

$$\begin{aligned} \Delta Z &= x[\bar{Z}(\eta) - \bar{Z}(0)] \\ &= -x \sum_{k=1}^{\infty} \epsilon_{2k} \eta^{2k} / 2k(2k-1) \end{aligned} \quad (21)$$

The lift and drag coefficient are respectively

$$C_L = \int_0^1 \Delta C_p(\eta) d\eta = \sum_{i=0}^{\infty} \epsilon_{2i} l_i$$

$$C_M = - \int_0^1 C_p(\eta) w(\eta) d\eta = \epsilon_0 C_L + \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} \epsilon_{2i} \epsilon_{2j} d_{ij} \quad (22)$$

where

$$l_i = \int_0^1 \Delta C_{p2i}(\eta) d\eta$$

$$d_{ij} = - \int_0^1 \Delta C_{p2i}(\eta) w_{2j}(\eta) d\eta \quad (23)$$

and  $\Delta C_{p2i}(\eta)$  is the pressure distribution due to the twist distribution  $w_{2i}(\eta) = \eta^{2i}/2i$ .

It can now be easily shown that

$$d_{ij} = -4 \tan \delta \sum_{m=0}^{\infty} \frac{J_{2m} a^{2m}}{2(m+i)+1} \sum_{k=0}^{m+i} J_{2k} G_{2(m+i-j-k)/2j}$$

$$l_0 = 4 \tan \delta / E(k)$$

$$l_i = 4 \tan \delta \sum_{m=0}^{\infty} \frac{J_{2m} a^{2m}}{2(m+i)+1} \sum_{k=0}^{m+i} J_{2k} G_{1(m+i-k)} \quad (24)$$

where

$$G_0 = \pi/4$$

$$G_{2p} = \int_0^1 (1-\eta^2)^{1/2} \eta^{2p} d\eta = \pi J_{2p} / 4(p+1) \quad (25)$$

For a conical delta wing the chordwise distribution of lift is linear hence the pitching moment coefficient about the wing apex is

$$C_M = -\frac{2}{3} C_L \quad (26)$$

#### Properties of Basic Shapes

We shall confine our attention to wings for which the attachment line is prescribed at the leading edge and accordingly put  $C_0 = 0$  wherever they appear in the above results. Also for our purposes, we shall define the  $p^{\text{th}}$  basic shape as one carrying a twist distribution  $w_{2p} = \epsilon_{0p} + \epsilon_{2p} \eta^{2p}/2p$  where  $\epsilon_{0p}$  and  $\epsilon_{2p}$  are related through the condition  $C_0 = 0$ .

The geometrical properties of the basic shapes are now easy to visualize. It may immediately be seen that for  $p > 0$ , the wing will have drooped leading edges and with increasing values

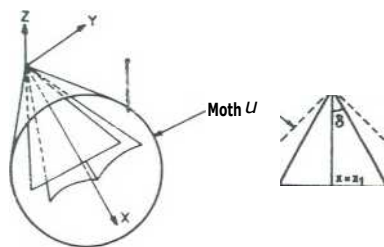


Fig. 1 The Wing-Mach Cone System

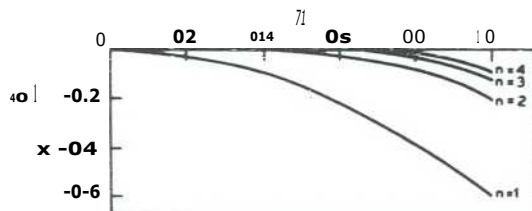
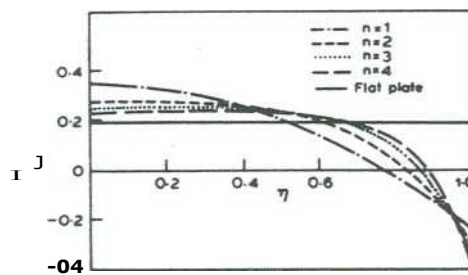
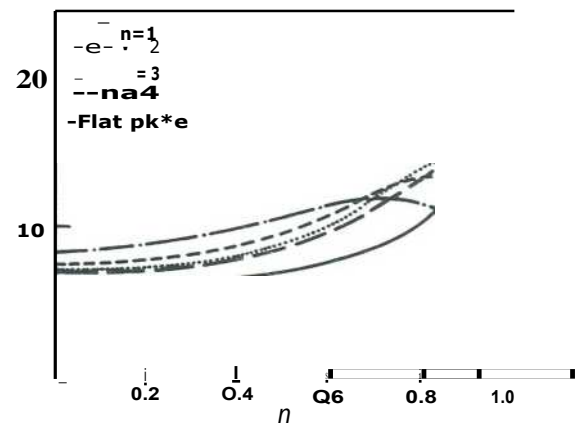
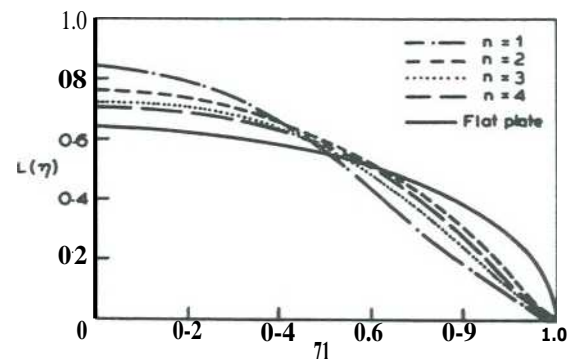
Fig. 2a Spanwise ordinate distribution ( $a=0.50$ )Fig. 2b Spanwise angle of attack distribution ( $a=0.50$ )Fig. 2c Spanwise pressure distribution ( $a=0.50$ )Fig. 2d Spanwise lift distribution ( $a=0.50$ )

TABLE I OVERALL AERODYNAMIC CHARACTERISTICS OF BASIC SHAPES

Wing P	$\frac{C_D \tan \delta}{C_L^2}$	$\frac{L}{D} \frac{C_L}{\tan \delta}$	$\frac{a_0 \tan \delta}{C_L}$	e	a
1	.109	9.165	.321	1.371	0.1
2	.098	10.153	.241	1.238	
3	.094	10.646	.215	1.180	
4	.091	10.940	.202	1.149	
Delta 1	.083	12.118	.162	1.034	
Delta 2	.162	6.170	.162	2.038	
1	.151	6.631	.350	1.895	0.5
2	.140	7.138	.273	1.760	
3	.135	7.381	.247	1.702	
4	.133	7.524	.233	1.670	
Delta 1	.124	8.076	.193	1.555	
Delta 2	.193	5.182	.193	2.421	
1	.182	5.480	.369	2.293	0.7
2	.172	5.807	.293	2.164	
3	.168	5.957	.268	2.109	
4	.165	6.044	.255	2.079	
Delta 1	.157	6.356	.214	1.974	
Delta 2	.214	4.670	.214	2.692	

Delta 1 - Plane delta wing with leading edge suction

Delta 2 - Plane delta wing without leading edge suction



TABLE 2 COMPARISON OF WINGS AT DIFFERENT VALUES OF 'a' FOR THE  
BASIC SHAPES  
p = 4

$\eta$	$\Delta C_p(\eta) / C_L$			$L(\eta)$		
	a = 0.1	a = 0.5	a = 0.7	a = 0.1	a = 0.5	a = 0.7
0.00	.707	.706	.704	.707	.706	.704
0.20	.726	.725	.723	.689	.688	.686
0.40	.796	.794	.791	.630	.629	.628
0.60	.982	.978	.971	.514	.514	.515
0.70	1.169	1.164	1.156	.422	.423	.425
0.80	1.437	1.435	1.430	.298	.300	.303
0.90	1.657	1.668	1.685	.140	.142	.145
0.95	1.529	1.550	1.584	.057	.058	.060
1.00	.000	.000	.000	.000	.000	.000
$\eta$	$\alpha(\eta) \tan \delta / C_L$			$\Delta Z(\eta) \tan \delta / x C_L$		
	a = 0.1	a = 0.5	a = 0.7	a = 0.1	a = 0.5	a = 0.7
0.00	.202	.233	.255	-.000	-.000	-.000
0.20	.202	.233	.255	-.000	-.000	-.000
0.40	.201	.233	.255	-.000	-.000	-.000
0.60	.189	.222	.245	-.002	-.002	-.001
0.70	.160	.196	.222	-.006	-.005	-.005
0.80	.080	.123	.158	-.017	-.016	-.014
0.90	-.110	-.049	.007	-.045	-.040	-.035
0.95	-.279	-.202	-.127	-.069	-.062	-.055
1.00	-.496	-.397	-.299	-.100	-.091	-.080

p = 2

$\eta$	$\Delta C_p(\eta) / C_L$			$L(\eta)$		
	a = 0.1	a = 0.5	a = 0.7	a = 0.1	a = 0.5	a = 0.7
0.00	.764	.760	.757	.764	.760	.757
0.20	.791	.788	.783	.738	.735	.731
0.40	.897	.891	.884	.651	.650	.648
0.60	1.115	1.109	1.099	.485	.487	.489
0.70	1.251	1.248	1.242	.369	.372	.376
0.80	1.350	1.356	1.363	.236	.239	.243
0.90	1.276	1.296	1.324	.098	.100	.103
0.95	1.045	1.068	1.104	.037	.038	.040
1.00	.000	.000	.000	.000	.000	.000
$\eta$	$\alpha(\eta) \tan \delta / C_L$			$\Delta Z(\eta) \tan \delta / x C_L$		
	a = 0.1	a = 0.5	a = 0.7	a = 0.1	a = 0.5	a = 0.7
0.00	.241	.273	.293	-.000	-.000	-.000
0.20	.240	.272	.293	-.000	-.000	-.000
0.40	.225	.258	.280	-.005	-.005	-.004
0.60	.159	.197	.225	-.027	-.025	-.023
0.70	.089	.133	.167	-.051	-.047	-.042
0.80	-.019	.034	.078	-.087	-.080	-.072
0.90	-.175	-.110	-.052	-.139	-.128	-.115
0.95	-.276	-.203	-.135	-.172	-.159	-.143
1.00	-.382	-.300	-.223	-.208	-.193	-.173

of  $p$  the central portion of the wing will become more and more flat. The droop becomes noticeable towards the outboard portion and rapidly increases as it nears the leading edge. Consequently, the spanwise surface slope will be low in the central portion of the wing but will rapidly increase as it progresses towards the leading edge. A similar trend will be shown by the incidence distribution, which will be positive at the root and continuously decrease from the root to the tip to satisfy the attachment condition. The droop and the washout in incidence near the leading edge will help the flow turn smoothly and hence may be expected to present properties closer to the real situation. The wings are also easy to manufacture and are free from kinks, Figs. 2a, b.

The aerodynamic properties are a little more difficult to visualise. From Table 1 and Figs. 2a-d, it is noticed that the wing properties depend strongly on the amount of spanwise curvature for a given lift. Thus with increasing curvature (increasing  $p$ ), the wing incidence  $\alpha_0$  decreases and moves towards the flat plate value while the suction peak moves outboard and increases in magnitude. The spanwise lift distribution tends towards the ideal elliptic distribution and hence the drag coefficient and the drag factor  $e = \pi AR C_D / C_L^2$  decreases and the lift to drag ratio ( $L/D$ ) increases. These effects are primarily due to the suction peak acting on the forward facing droop which produces a thrust. For  $p=4$ , the  $C_D$  is substantially close to the plane delta with suction. For a given  $p$  it approaches the flat plate drag faster for increasing values of  $a$ .

The spanwise lift distribution, unlike the plane delta which has an infinite slope at the leading edge, has zero slope; the magnitude of the lift increases at the centre and decreases outboard near the leading edge in comparison to the plane delta. The effect is progressively less pronounced with increasing  $p$ .

This behaviour is to be expected since the angle of attack decreases from the root to the tip and therefore the wing tip is less loaded compared to a plane delta wing. This gives better tip stall characteristics, and lesser bending moment at the wing root.

Conically cambered wings may show some advantages over other shapes under off-design conditions. Thus at any supersonic speed with the leading edges subsonic, the load will vanish at some incidence and, therefore, the attachment line is unlikely to lie on the upper surface for part of the wing and on the lower surface for

the remainder, even away from the design incidence and Mach number.

From Table 2 an interesting fact is observed. For a given  $p$ , the aerodynamic and geometric properties do not vary much for wings designed for different values of the slenderness parameter,  $a$ . This is an indication that a wing designed for a given apex angle and Mach number will behave almost as well at off-design Mach numbers.

Finally, from Eq.(1) it is noticed that the pressure distribution will remain the same if the term  $(1-a^2\eta^2)^{-1/2} dw/d\eta$  in the integral remain the same. Hence for a given pressure distribution we can find the downwash distribution at a given Mach number and apex angle. This is particularly easy in our case since  $w$  is expressed in even powers of  $\eta$  and  $(1-a^2\eta^2)^{-1/2}$  may be expanded in similar powers for  $a < 1$  and the results of the previous section applied.

### Conclusions

A class of conically cambered delta wings with subsonic leading edges and a polynomial twist distribution of even ordered term is studied analytically. The results are found to be simpler than any other available in the literature. It also has an added advantage of easy visualisation of its aerodynamic and geometric properties.

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